AIAA 95-1327

Multi-Criterion Preliminary Design of a Tetrahedral Truss Platform

K. Chauncey Wu
NASA Langley Research Center
Hampton, Virginia

36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference

April 10-13, 1995 / New Orleans, LA

MULTI-CRITERION PRELIMINARY DESIGN OF A TETRAHEDRAL TRUSS PLATFORM

K. Chauncey Wu*
NASA Langley Research Center
Hampton, VA 23681-0001

Abstract

An efficient method is presented for multi-criterion preliminary design and demonstrated for a tetrahedral truss platform. The present method requires minimal analysis effort and permits rapid estimation of optimized truss behavior for preliminary design. A 14m-diameter, 3-ring truss platform represents a candidate reflector support structure for space-based science spacecraft. The truss members are divided into 9 groups by truss ring and position. Design variables are the cross-sectional area of all members in a group, and are either 1, 3 or 5 times the minimum member area. Non-structural mass represents the node and joint hardware used to assemble the truss structure. Taguchi methods are used to efficiently identify key points in the set of Pareto-optimal truss designs. Key points identified using Taguchi methods are the maximum frequency, minimum mass, and maximum frequency-to-mass ratio truss designs. Low-order polynomial curve fits through these points are used to approximate the behavior of the full set of Pareto-optimal designs. The resulting Pareto-optimal design curve is used to predict frequency and mass for optimized trusses. Performance improvements are plotted in frequency-mass (criterion) space and compared to results for uniform trusses. Application of constraints to frequency and mass and sensitivity to constraint variation are demonstrated.

Introduction

Design of aerospace systems inevitably requires simultaneous satisfaction of many different, and usually conflicting, criteria.[1] Detailed information about some configuration which meets the requirements is typically not required at a preliminary design level; instead, confirmation that an acceptable design exists is sufficient. Various techniques have been developed for solution of multi-criterion optimization problems.[2] In one common method, a single scalar objective function is formed from the weighted sum of the criteria of interest and used for optimization analyses. By varying the relative weights assigned to each criterion, the set of Pareto-optimal configurations may be found.[3] A configuration is called a Pareto optimum if no improvement may be made in one criterion without simultaneous degradation of at least one of the other criteria.[4] In this study, an efficient method is presented for predicting the frequency and mass of Pareto-optimal designs of a tetrahedral truss platform.

Several proposed Earth-science and deep-space astrophysics spacecraft (fig. 1) require large, highly-accurate truss platforms on the order of 10 meters in diameter or larger[5] to support faceted reflector surfaces. A lattice truss, which has inherently high natural frequencies and low mass, is a likely candidate for the primary mirror support structure. A 14m-diameter, 3-ring tetrahedral truss platform, shown in figure 2a, is representative of the reflector support structures described previously. Previous design studies of this structure[6] used natural frequency as the sole optimization criterion. In the present study, the criteria of interest are the natural frequency and mass of the truss. Trusses which have been simultaneously optimized for high frequency and low mass form the set of Pareto-optimal designs.

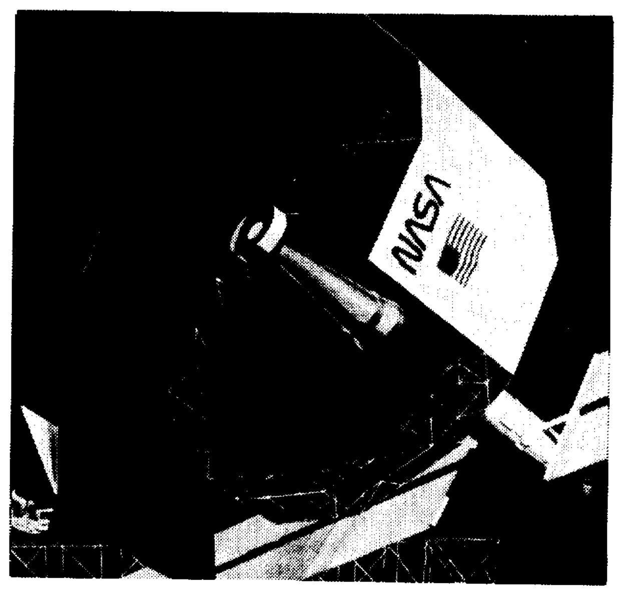

Figure 1. 20m-diameter high-precision reflector.

*Aerospace Engineer, Structures Division.

The 315 members in the truss are subdivided into 9 member groups by ring number and relative position in the truss. The cross-sectional area of all members in any group are equal, and the set of member areas for all groups are the design variables. Each design variable is either 1, 3 or 5 times the minimum area. Therefore, there are 3^9 or 19,683 possible truss configurations in the entire design space. A search through design space for the set of Pareto-optimal designs would be both tedious and time-consuming. Instead, a simplified process is presented here in which certain key designs from the Pareto-optimal set are determined, and the behavior of the remaining Pareto-optimal designs is inferred from the behavior of the key designs.

The three key designs chosen have maximum frequency, minimum mass, and maximum frequency-to-mass ratio. Approximate solutions for these designs are determined with Taguchi methods and results are plotted in frequency-mass (criterion) space. Behavior of the full set of Pareto-optimal designs is approximated by low-order polynomial curves which are fitted through the three key design points in criterion space. Finally, a trade study is performed to illustrate the use of the frequency-mass relationship for the Pareto-optimal trusses in preliminary design.

Truss Geometry and Finite Element Analysis

The tetrahedral truss configuration evaluated in this study is comprised of 315 truss members, each 2 meters in length. A perspective view of the truss platform and a repeating cell are shown in figure 2a. The truss, which has a diameter of 14 meters across corners and a depth of 1.63 meters, is shown sub-

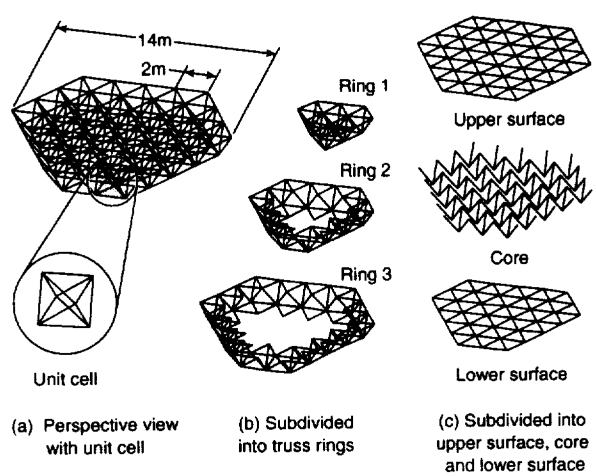

Figure 2. 3-ring tetrahedral truss platform.

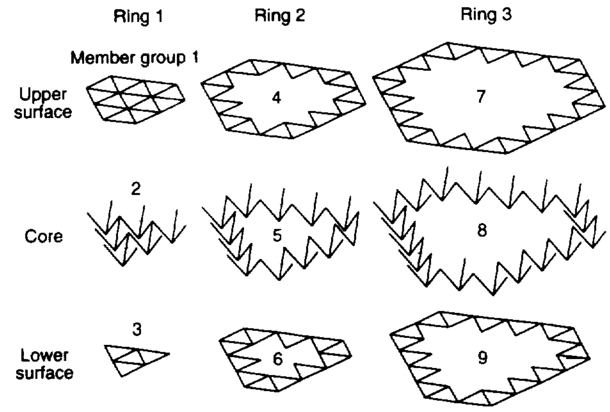

Figure 3. 3-ring truss member groups.

divided into 3 circumferential truss rings in figure 2b, and partitioned into upper surface, core, and lower surfaces in figure 2c. The truss subdivisions in figures 2b and 2c are used to define the 9 member groups shown in figure 3. All of the members in a given group have the same cross-sectional area; the design variables for this problem are the cross-sectional areas associated with each member group.

A commercial finite element code[7] is used to construct a linear analysis model which has pinned-end axial stiffness elements. The truss members are assumed to be comprised of a composite tube with a fixed outer diameter of 3.18×10^{-2} meters, and a joint at each end which attaches to a truss node. The minimum cross-sectional area of the truss members is 6.45×10^{-5} m^2, which is assumed to be sufficient to satisfy minimum gage and member buckling requirements. The truss members are assumed to have an elastic modulus of 1.23×10^{11} Pa and mass density of 1348 kg/m^3, nominal properties for a high-performance graphite-epoxy material. Nonstructural mass is included to represent the node and joint hardware used to assemble the truss structure. Each truss node is represented by a 0.39 kg point mass and each truss joint is represented by a 0.21 kg point mass. A total of 162.41 kg of nonstructural mass is distributed among the 84 truss nodes. An eigenvalue analysis is performed to compute the lowest flexible-body frequency of the truss with free-free boundary conditions. The mode shape associated with the fundamental frequency is asymmetric, anticlastic bending of the truss.[6]

Optimization Criteria

A high truss natural frequency, f, is typically desirable to maintain adequate separation between structural and attitude control system frequencies. Also, high natural frequencies result in lower dynamic amplitudes and faster damping of transient disturbances, so stringent spacecraft pointing and surface accuracy requirements for science missions may be met. Thus, it is particularly important to maintain high natural frequency for trusses used in precision reflectors, where the panel support structure must be both stiff and accurate.[8]

The truss mass, m, is another criterion for structural optimization problems. Low mass is necessary because of the extremely high cost of transportation to orbit, with launch costs for current systems on the order of 10,000 dollars per kilogram.[9] In addition, high masses are often reflected in higher costs for additional material, fabrication and processing. In this study, both frequency and mass are optimization criteria used in determining the Pareto-optimal designs.

Pareto-Optimal Design Curve

In this section, characteristics and construction of a Pareto-optimal design curve are discussed for optimization criteria of frequency and mass. The Pareto-optimal design curve is used to approximate the behavior of the actual set of Pareto-optimal designs in criterion space. As the design parameters are varied, the frequency and mass of the structure are assumed to vary within certain limits. These limits can be graphically determined by plotting frequency and mass for all possible designs. The shaded region in the criterion space plot of figure 4 represents such a plot of all possible configurations for a given design problem. Since designs which combine high frequency and low mass are desired, the

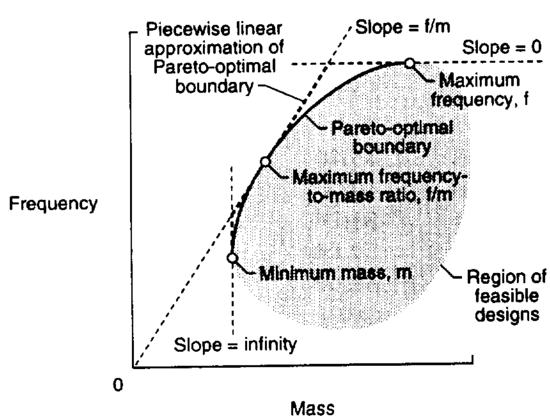

Figure 4. Criterion-space plot of feasible designs and Pareto-optimal boundary.

upper left-hand boundary of this region of feasible designs is identified as the Pareto-optimal boundary, and is shown in the figure as a dark solid line. Thus, the Pareto-optimal design curve is an approximation to this maximum-frequency, minimum-mass boundary of the region of feasible designs. Any deviation from this boundary into the region of feasible designs will result in a decrease in frequency, an increase in mass, or both.

Three key points, the maximum frequency design, the minimum mass design, and the maximum frequency-to-mass ratio design, are identified on the Pareto-optimal boundary in figure 4. The maximum frequency design and the minimum mass design are important because they are the end points of the Pareto-optimal boundary. The maximum frequency design is the upper limit of the Pareto-optimal boundary, and the slope through this point must be zero (no feasible designs exist above this point). Similarly, the minimum mass design defines the left-hand limit of the Pareto-optimal boundary, and the slope through this point must be infinite (no designs exist to the left of this point). Finally, the upper, left-hand limit for the Pareto-optimal boundary is assumed to be a line through the origin and the maximum frequency-to-mass ratio design. Thus, the slope through the maximum frequency-to-mass ratio design point is equal to $(f - 0)/(m - 0)$, or f/m.

A simple, conservative approximation of the Pareto-optimal boundary consists of the three bounding lines described previously. This piecewise linear design curve is shown in figure 4 as a dark dashed line. Less conservative approximations of the design curve can be constructed from low-order polynomial curve fits through the key points and their slopes described previously. The accuracy of such curve fits depends on the behavior of the actual Pareto-optimal boundary, which is assumed to be both smooth and continuous in this study.

Computation of Key Truss Designs

To determine the Pareto-optimal design curve for the truss example, Taguchi methods[10] are used to identify the maximum frequency, minimum mass, and maximum frequency-to-mass ratio truss designs. Taguchi methods are an unconstrained optimization technique which use an orthogonal array from design-of-experiments theory to reduce the computational effort necessary to locate optimized truss designs. An array with 27 rows, shown in table 1, is selected using the program of reference 11. In any pair of columns of an orthogonal array, every possible combination of array elements occurs, and occurs an equal number of times. Also, if the array elements 1, 3 and 5 are replaced by -1, 0 and +1, then each column is linearly independent from every other column, with a scalar product of zero.

The design variables, which are the cross-sectional areas of the 9 member groups in figure 3, are assigned to the 9 columns of the orthogonal array in table 1. Allowable values for the design variables are 1, 3 and 5 times the minimum area of 6.45×10^{-5} m^2. Thus, the array elements in table 1 are defined as the normalized member area A, or the ratio of the member area to the minimum area, where A = 1, 3 or 5. Truss configurations are presented in the form (123 456 789), where each value in parentheses is the normalized area for member groups 1 through 9. If every possible combination of design variables and areas were run, 3^9 or 19,683 analyses, collectively termed the "full-factorial" set, would be required. In the present application of Taguchi methods, only 27 analyses are required to determine an optimized truss configuration. Note that each of the 19,683 combinations is assumed to be a feasible design since the optimization method is unconstrained. Thus, the values of the design variables must be selected with manufacturing and logistical considerations in mind. For example, use of a composite material for the truss members restricts the area to discrete values due to the integer number of lamina in the tube wall.

Each row of the array in table 1 represents an analysis case which must be performed. In each case, the design variables are set to the values indicated in the row of the orthogonal array, and the frequency, mass and frequency-to-mass ratio are computed using the vibrational analysis described previously. The values of these objective functions are shown in table 1 for each of the 27 cases. For example, the truss configuration in analysis case 25 has normalized areas of (551 513 135), and frequency, mass, and frequency-to-mass ratio of 44.20

Table 1: Orthogonal array and structural analysis results for Taguchi analysis

Case	Normalized area A for truss member group									f, Hz	m, kg	f/m, Hz/kg
	1	2	3	4	5	6	7	8	9			
1	1	1	1	1	1	1	1	1	1	29.93	217.20	0.1378
2	1	1	3	5	5	5	5	5	5	39.94	404.01	0.0989
3	1	1	5	3	3	3	3	3	3	38.13	315.30	0.1209
4	1	3	1	5	5	5	3	3	3	39.61	351.83	0.1126
5	1	3	3	3	3	3	1	1	1	36.19	263.12	0.1375
6	1	3	5	1	1	1	5	5	5	31.36	340.35	0.0921
7	1	5	1	3	3	3	5	5	5	36.84	376.87	0.0978
8	1	5	3	1	1	1	3	3	3	32.03	288.16	0.1112
9	1	5	5	5	5	5	1	1	1	38.20	309.04	0.1236
10	3	1	1	5	3	1	5	3	1	31.58	323.65	0.0976
11	3	1	3	3	1	5	3	1	5	39.31	316.34	0.1243
12	3	1	5	1	5	3	1	5	3	38.39	321.56	0.1194
13	3	3	1	3	1	5	1	5	3	42.72	320.52	0.1333
14	3	3	3	1	5	3	5	3	1	34.92	328.87	0.1062
15	3	3	5	5	3	1	3	1	5	36.97	330.95	0.1117
16	3	5	1	1	5	3	3	1	5	35.46	326.78	0.1085
17	3	5	3	5	3	1	1	5	3	40.00	335.13	0.1194
18	3	5	5	3	1	5	5	3	1	38.74	337.21	0.1149
19	5	1	1	3	5	1	3	5	1	32.48	329.91	0.0985
20	5	1	3	1	3	5	1	3	5	41.58	322.60	0.1289
21	5	1	5	5	1	3	5	1	3	39.06	334.08	0.1169
22	5	3	1	1	3	5	5	1	3	36.67	329.91	0.1112
23	5	3	3	5	1	3	3	5	1	39.71	338.26	0.1174
24	5	3	5	3	5	1	1	3	5	41.16	337.21	0.1221
25	5	5	1	5	1	3	1	3	5	44.20	336.17	0.1315
26	5	5	3	3	5	1	5	1	3	35.09	344.52	0.1019
27	5	5	5	1	3	5	3	5	1	39.50	343.48	0.1150

Table 2: Predicted optimal truss configurations

Table 2a: Average frequency, Hz

A	Truss member group 1	2	3	4	5	6	7	8	9
1	35.80	36.71	36.61	35.54	37.45	34.51	**39.15**	36.32	35.69
3	37.56	37.70	37.64	37.85	**37.49**	38.10	37.02	**37.99**	37.97
5	**38.83**	**37.78**	**37.94**	**38.81**	37.25	**39.59**	36.02	37.88	**38.53**

Table 2b: Average mass, kg

A	Truss member group 1	2	3	4	5	6	7	8	9
1	**318.43**	**320.52**	**323.65**	**313.21**	**314.25**	**316.34**	**306.95**	**307.99**	**310.08**
3	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78	326.78
5	335.13	333.04	329.91	340.35	339.30	337.22	346.61	345.57	343.48

Table 2c: Average frequency-to-mass ratio, Hz/kg

A	Truss member group 1	2	3	4	5	6	7	8	9
1	0.1147	0.1159	0.1143	0.1145	**0.1199**	0.1102	**0.1282**	**0.1193**	**0.1165**
3	0.1150	**0.1160**	**0.1162**	**0.1168**	0.1155	0.1173	0.1133	0.1162	0.1163
5	0.**1159**	0.1137	0.1152	0.1144	0.1102	**0.1181**	0.1041	0.1102	0.1128

Hz, 336.17 kg, and 0.1315 Hz/kg. The frequency, mass, and frequency-to-mass ratio data from table 1 are reduced into tables 2a, 2b, and 2c. Each table entry is the average of the objective function values for a given design variable (1 to 9) at a given value (1, 3 or 5). For example, the 35.80 Hz average frequency from table 2a for member group 1 and normalized area 1 is the average of the computed frequencies for all cases in which member group 1 is assigned a normalized area of 1 (cases 1 through 9 from table 1).

The optimized truss design is determined by setting the cross-sectional area of each of the 9 design variables to the value (1, 3 or 5) which yields the best results (shown in boldface in table 2) of the three allowable values. Thus, for the maximum frequency truss design, member group 1 will have a normalized member area of 5, since the corresponding average frequency of 38.83 Hz from table 2a is higher than the other frequencies in that column. The predicted maximum frequency truss configuration, shown in table 2a, has normalized member areas of (555 535 135). This design resembles a tapered plate, where the thickness at the perimeter is less than the thickness in the center. The minimum-mass truss configuration, shown in table 2b, is one where every member group has the lowest possible normalized area of 1. Since the mass of a member is proportional to its area, this result makes physical sense. The predicted truss configuration with the maximum frequency-to-mass ratio, shown in table 2c, has normalized areas of (533 315 111). This truss is similar to a tapered honeycomb sandwich, with stiff face sheets in truss rings 1 and 2, minimum-mass face sheets in ring 3 and a low-mass core throughout.

Table 3: Optimized and uniform truss performance

Description	Configuration	f, Hz	m, kg	f/m, Hz/kg
Baseline uniform, A=3	333 333 333	42.96	326.78	0.1315
Maximum frequency	555 535 135	50.73	365.39	0.1388
percent from baseline		+18.07	+11.82	+5.55
Minimum mass, A=1	111 111 111	29.93	217.20	0.1378
percent from baseline		-30.33	-33.53	+4.80
Maximum frequency-to-mass ratio	533 315 111	42.77	277.73	0.1540
percent from baseline		-0.45	-15.01	+17.11
Uniform, A=5	555 555 555	48.40	436.36	0.1109

The actual performance of each optimized configuration is determined from a vibrational analysis where the design variables are set to their predicted values in table 2. For example, finite element analysis of the maximum frequency configuration shows that this design has a fundamental frequency of 50.73 Hz and a mass of 365.39 kg. The three optimized truss configurations are compared to the baseline design (where every member group has a normalized area of 3) in table 3. Also shown in table 3 are results for the uniform truss where every member has the maximum normalized area of 5. The data from table 3 are plotted in criterion space in figure 5.

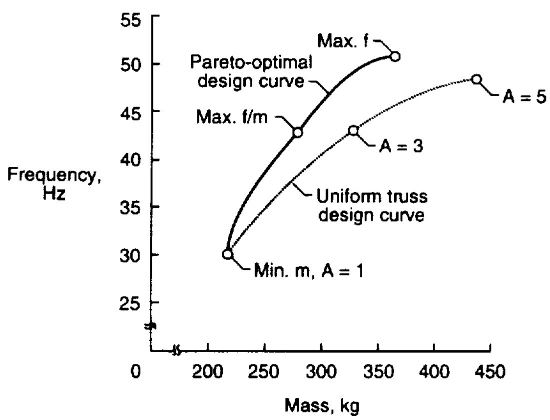

Figure 5. Design curves for Pareto-optimal and uniform trusses.

Computation of Pareto-Optimal Design Curve

Low-order polynomial curve fits are made to the optimized truss configurations in table 3 to determine the Pareto-optimal design curve. The slope constraints discussed previously (zero slope at the maximum frequency design, infinite slope at the minimum mass design, and slope f/m at the maximum frequency-to-mass ratio design) are applied by differentiation of the polynomial equations. A third-order polynomial with the general form

$$f = \alpha + \beta m + \gamma m^2 + \delta m^3 \qquad (1)$$

is chosen to represent the design curve between the maximum frequency and maximum frequency-to-mass ratio designs in criterion space. An equation for the slope in this region is given by differentiation of equation 1 with respect to m, resulting in

$$df/dm = 0 + \beta + 2\gamma m + 3\delta m^2 \qquad (2)$$

To determine the unknown polynomial coefficients α, β, γ, and δ, a matrix equation

$$\begin{bmatrix} 1 & m & m^2 & m^3 \\ 1 & m & m^2 & m^3 \\ 0 & 1 & 2m & 3m^2 \\ 0 & 1 & 2m & 3m^2 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{Bmatrix} = \begin{Bmatrix} f \\ f = f_{max} \\ df/dm = f/m \\ df/dm = 0 \end{Bmatrix} \begin{matrix} \text{(max. f/m design)} \\ \text{(max. f design)} \\ \text{(slope at max. f/m)} \\ \text{(slope at max. f)} \end{matrix} \qquad (3)$$

is assembled and evaluated with the program of reference 12. The rows of equation 3 correspond to the maximum frequency-to-mass ratio and maximum frequency truss designs and their slopes. Since there are four unknown coefficients, an exact cubic fit is made through the two points in criterion space. Substitution of the appropriate values from table 3 and solution of equation 3 gives

$$f = 45.6626 - 0.4520m + 0.002588m^2 - 0.000003593m^3 \qquad (4)$$

Because the slope at the minimum mass design is infinite, equation 1 cannot be used for the design curve between the minimum mass and maximum frequency-to-mass ratio designs. However, the reciprocal dm/df of the slope is zero, which means that an equation of the form

$$m = \varepsilon + \zeta f + \eta f^2 + \theta f^3 \qquad (5)$$

can be used to approximate the design curve between the minimum mass and maximum frequency-to-mass ratio designs. Equation 5 is solved for the polynomial coefficients ε, ζ, η, and θ as described previously, resulting in an equation

$$m = 1228.1303 - 83.4996f + 2.1941f^2 - 0.01780f^3 \qquad (6)$$

between the minimum mass and maximum frequency-to-mass ratio designs. The third-order curves (defined by equations 4 and 6) are shown in the plot of figure 5. This pair of curves is used to predict the trade-off between frequency and mass for the Pareto-optimal truss designs.

Verification of Pareto-Optimal Design Curve

To determine the accuracy of the Pareto-optimal design curve, frequency and mass were computed for the 19,683 full-factorial cases. Criterion space results of the full-factorial analyses are shown with the design curve in figure 6. Although the design curve shown is not the actual boundary for the full-

Table 4: Predicted and global optimized trusses

Criterion	Configuration	f, Hz	m, kg	f/m, Hz/kg
Maximum frequency				
Predicted	555 535 135	50.73	365.39	0.1388
Global	555 555 133	51.19	361.22	0.1417
Percent error		+0.90	-1.15	+2.05
Minimum mass				
Predicted	111 111 111	29.93	217.20	0.1378
Global	111 111 111	29.93	217.20	0.1378
Percent error		0.00	0.00	0.00
Maximum frequency-to-mass ratio				
Predicted	533 315 111	42.77	277.73	0.1540
Global	515 335 111	46.23	287.12	0.1610
Percent error		+7.48	+3.27	+4.35

factorial data, the majority of the data are bounded by the cubic curve fits from equations 4 and 6. The trusses which have the highest frequency, lowest mass and highest frequency-to-mass ratio of the 19,683 full-factorial configurations were extracted and are compared to corresponding results from the Taguchi analysis in table 4. Agreement between results for the Taguchi and full-factorial analyses is excellent for the maximum frequency and minimum mass trusses. Results for the maximum frequency-to-mass ratio trusses show an error of under 5 percent in the frequency-to-mass ratio.

Two key assumptions are necessary for the design method in this study to be valid. The first assumption is that a low-order polynomial is sufficient to define the Pareto-optimal design curve. Examination of figure 6 shows that, for this problem, a cubic polynomial curve provides a reasonable upper bound to the full-factorial data. However, this is not true for every design problem. The present method is applicable to two of the three analysis cases in reference 3, which have smooth, continuous Pareto-optimal boundaries. The boundary of the third case in reference 3 is highly discontinuous, and the present method would not provide a good approximation of the boundary. The second assumption is that every point on the Pareto-optimal design curve has a corresponding feasible design. Although this assumption may be true when the design variables vary continuously between limits (as in calculus-based optimization), it is not the case in this example, where the design variables take on only discrete values. *Thus, not every point on the design curve in figure 6 has an associated configuration in the full-factorial data; however, many feasible designs are very close to the design curve. Note that the design curve bounds the region of feasible designs, but does not tell the designer what the values of the design variables are for trusses along that boundary.

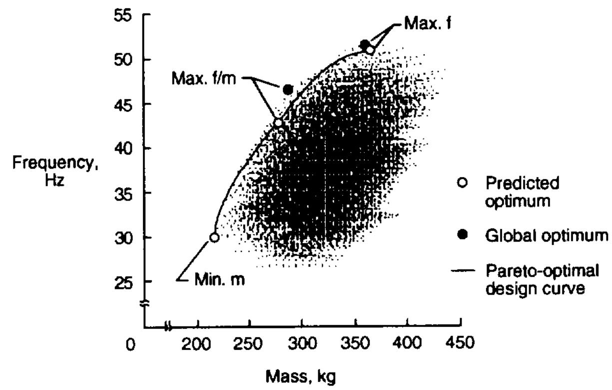

Figure 6. Full-factorial data and Pareto-optimal design curve.

Multi-Criterion Preliminary Design of a Truss Platform

The first step in preliminary design is to determine the performance improvements that can be achieved through optimization. Such estimates can be made by comparison of the uniform truss performance to that of the Pareto-optimal structures predicted by equations 4 and 6. To provide a design curve for uniform trusses, a second-order curve fit (equation 1 with $\delta = 0$) is made to the uniform truss frequency and mass data from table 3. This curve fit is exact since there are three uniform truss designs (A = 1, 3 and 5) and three unknowns. Solution for the polynomial coefficients gives

$$f = -18.3359 + 0.2909m - 0.0003162m^2 \quad (7)$$

The uniform truss design curve from equation 7 is plotted in figure 5 with the Pareto-optimal design curve. Note that a semi-empirical equation for uniform truss frequency is presented in reference 6, which may be used instead of equation 7. The (111 111 111) configuration is common to both the uniform and Pareto-optimal design curves, since it is both a uniform truss with A = 1 and the minimum mass design.

With the exception of the minimum mass truss design, there are significant differences between the behavior of the Pareto-optimal and uniform truss design curves. In general, the performance gains from optimization will depend on additional constraints which may be imposed on the design. For illustrative purposes, two constraint values, a 40 Hz lower limit on frequency and a 300 kg upper limit on mass, are selected for this problem and shown in figure 7. Computation of frequency and mass for these constraints using equations 4 and 6 shows that use of an optimized truss will increase the truss frequency by 5.49 Hz (13.56 percent) over a uniform truss of equal mass, and an optimized truss will reduce the truss mass by 40.17 kg (13.60 percent) over a uniform truss with equal frequency. The shaded region labeled "feasible designs" in figure 7 contains all truss designs which satisfy the frequency and mass constraints and are bounded by the Pareto-optimal and uniform truss design curves. Note that none of the three uniform trusses which were analyzed satisfies the constraints, but other uniform trusses (possibly with non-integer normalized areas) may be located which do satisfy the constraints.

In preliminary design problems, the constraint values are often ill-defined, but nominal values must still be used for trade studies and performance estimates. The Pareto-optimal design curve may be used to predict the sensitivity of one criterion to variation in another criterion. For example, if the uncertainty in the mass constraint is ±5 percent (300 kg ± 15 kg), then equation 4 may be used to determine the resulting frequency variation from the nominal 45.97 Hz frequency. For a 5 percent reduction in the mass constraint (to 285 kg), the frequency is reduced by 4.55 percent (to 43.88 Hz). An increase of 5 percent in the truss mass (to 315 kg) results in an increase in frequency of 3.92 percent (to 47.77 Hz). These results are shown graphically in figure 8.

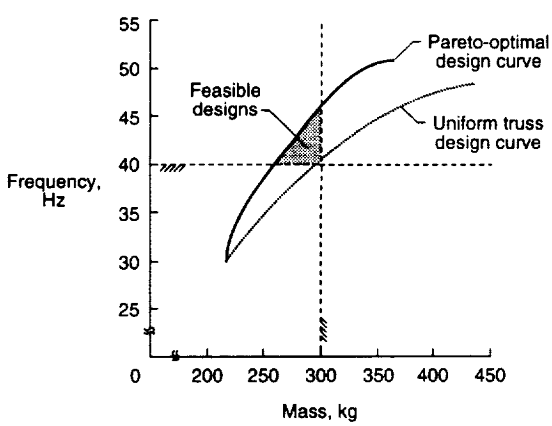

Figure 7. Pareto-optimal and uniform design curves with constraints.

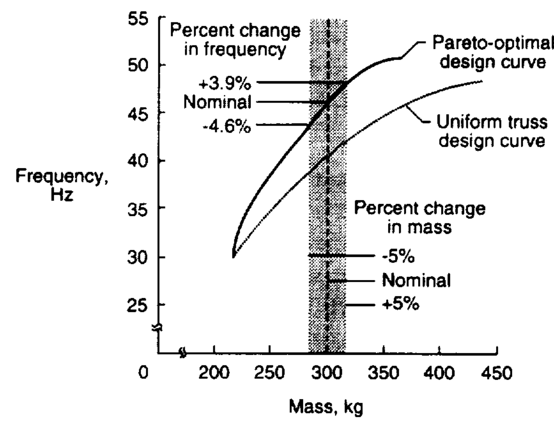

Figure 8. Frequency sensitivity for mass constraint variation.

One further aspect of multi-criterion preliminary design must still be addressed: selection of a nominal design. That is, of the feasible designs in figure 7, which one should be chosen for later studies? To answer this question, two additional considerations must be examined. First, the criteria must be prioritized in terms of their relative importance. In this truss design problem, is high frequency more important than low mass, or vice versa? Selection of one overriding criterion implies that the other criterion (or criteria) can be treated as a constraint. The original multi-criterion problem now resembles a typical single-criterion problem, where one criterion is considered dominant, and all other considerations are handled with constraints in the optimization. The difference lies in the fact that in the present method, the constraints are not imposed until after the Pareto-optimal designs are located in criterion space, instead of being imposed during optimization. The second consideration requires estimation of the constraint variations from experience or historical data. For example, is 5 percent growth in the 300 kg mass constraint likely, or is 10 percent a more realistic value? What is the resulting variation in frequency? What is the expected variation in the 40 Hz frequency constraint, and the associated mass variation? Selection of a design whose behavior is least sensitive to variations in the constraints would be most prudent.

Concluding Remarks

An efficient method is presented here for multi-criterion preliminary design, and demonstrated for a 14m-diameter, 3-ring tetrahedral truss platform. This method permits rapid estimates of the truss natural frequency and mass, the criteria of interest, to be made during preliminary design studies. The method is based on construction of an Pareto-optimal design curve, generated from only 31 analyses (27 cases from the orthogonal array, verification of 2 optimized designs and 2 uniform trusses), out of a possible 19,683 designs. The multi-criterion design method presented here does not depend on which optimization technique is used to generate the optimal frequency, mass and frequency-to-mass ratio truss designs, however Taguchi methods are both accurate and efficient for this problem. The full-factorial data are used to demonstrate the validity of assumptions made on the nature of the Pareto-optimal boundary.

The present method does not identify what specific combination of design variables will return the values of the criteria. Instead, the method indicates that a configuration which satisfies the given constraints may exist, which is probably more important during a preliminary design phase than details of a specific configuration. The ability to estimate criterion sensitivity to variation of constraints is very important in preliminary design problems, when constraint values are often ill-defined and may also change rapidly. Extension of this method to include additional criteria, such as cost, should be fairly straightforward, but physical insight may be lost for problems which have more than three criteria. The method discussed here has been successfully applied to optimal design of a truss platform for maximum frequency and minimum mass, and may also be applicable to other design problems where conflicting criteria must be satisfied.

References

1. Grooms, H. R.; DeBarro, C. F.; and Paydarfar, S.: *What is an Optimal Spacecraft Structure?* AIAA Journal of Spacecraft and Rockets, vol. 29, no. 4, July-August 1992, pp. 480-483.

2. Duckstein, L.: *Multiobjective Optimization in Structural Design: The Model Choice Problem.* In E. Atrek; R. H. Gallagher; K. M. Ragsdell; and O. C. Zienkiewicz (eds.), New Directions in Optimum Structural Design, John Wiley & Sons, Ltd., 1984, pp. 459-481.

3. Rakowska, J.; Haftka, R. T.; and Watson, L. T.: *Tracing the Efficient Curve for Multi-Objective Control-Structure Optimization.* Computing Systems in Engineering, vol. 2, no. 5-6, 1992, pp. 461-471.

4. Koski, J.: *Multicriterion Optimization in Structural Design.* In E. Atrek; R. H. Gallagher; K. M. Ragsdell; and O. C. Zienkiewicz (eds.), New Directions in Optimum Structural Design, John Wiley & Sons, Ltd., 1984, pp. 483-503.

5. Swanson, P. N.: A *Lightweight Low Cost Large Deployable Reflector (LDR); A Concept Study by the Jet Propulsion Laboratory.* JPL D-2283, June 1985.

6. Wu, K. C.; and Lake, M. S.: *Natural Frequency of Uniform and Optimized Tetrahedral Truss Platforms.* NASA TP-3461, November 1994.

7. Whetstone, W. D.: *EISI-EAL Engineering Analysis Language – Interim Release, EAL/325 User Instructions, Version 325.05.* Engineering Information Systems, Inc., January 1990.

8. Bush, H. G.; Herstrom, C. L.; Heard, W. L., Jr.; Collins, T. J.; Fichter, W. B.; Wallsom, R. E.; and Phelps, J. E.: *Design and Fabrication of an Erectable Truss for Precision Segmented Reflector Application.* AIAA Journal of Spacecraft and Rockets, vol. 28, no. 2, March-April 1992, pp. 251-257.

9. Keith, E. L.: *Low-Cost Space Transportation: The Search for the Lowest Cost.* Presented at the AAS/AIAA Spaceflight Mechanics Meeting, Johnson Space Center, Houston, TX, February 13, 1991. Available as AAS Paper 91-169.

10. Phadke, M.: *Quality Engineering Using Robust Design.* Prentice Hall, 1989.

11. Anon.: *JMP – Statistics for the Apple Macintosh, User's Guide, Version 3.* SAS Institute, Inc., January 1994.

12. Wolfram, S.: *Mathematica – A System for Doing Mathematics by Computer.* Addison-Wesley Publishing Company, Inc., 1988.